Setting a Good Example: Teachers' Choice of Examples and their Contribution to Effective Teaching of Numeracy

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This paper reports on teachers' choice of examples and the role they play in students' construction of knowledge. Selecting an appropriate example is a challenging task for teachers, with both the teacher's content and pedagogical content knowledge being a determining factor in the selection process. A case study approach was used to document the nature of three different teachers' choice of examples. Qualitative descriptions illustrate the types of examples selected and the understandings the students constructed from these examples. The findings indicate that teachers need to consider carefully their choice of examples to avoid the likelihood of students forming misconceptions about important mathematical concepts.

Background

According to Askew (2005) effective teaching of numeracy involves helping students acquire knowledge of and facility with numbers, number relations, and number operations and assisting them with building an integrated network of understanding, techniques, strategies and application skills. In assisting students to construct understanding, teachers often select examples to illustrate particular principles, concepts and techniques. The selection of examples can be an indicator of effective teaching for numeracy, with both the teacher's content and pedagogical content knowledge being a determining factor in the selection process. There has been considerable research into what constitutes effective teaching of numeracy (Groves, Mousley, & Forgasz, 2006) including the Effective Teaching of Numeracy project (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997a), which identified effective teachers of numeracy based on rigorous evidence of increases in pupil attainment, not on presumptions of "good practice". Their findings identified a number of characteristics which were common among effective numeracy teachers. Other studies (e.g., Jones, Tanner, & Treadaway, 2000; Clarke & Clarke, 2002; Saunders, 2004) supported these findings which indicated that effective teachers of numeracy:

- Maintained a focus on and taught for conceptual understanding of important mathematical ideas
- Used a variety of teaching approaches which foster connections between both different areas of mathematics and previous mathematical experiences
- Encouraged purposeful discussion through the use of question types to probe and challenge children's thinking and reasoning and encouraging children to explain their mathematical thinking
- Possessed knowledge and awareness of conceptual connections between the areas which they taught of the primary mathematics curriculum and confidence in their own knowledge of mathematics

Based on the commonalities identified among the various studies, the author devised a set of six "Principles of Practice" which provided part of the theoretical framework for conducting the study and involved the teacher's ability to: make connections, challenge all pupils, develop conceptual understanding, focus on the key ideas of mathematics, engage the students in purposeful discussion, and possess a positive attitude towards mathematics. This type of teaching places high demands on teachers' subject matter and pedagogic content knowledge (Commonwealth of Australia, 2004) and this became evident when particular teaching behaviours were examined. The author termed these behaviours, "observable numeracy practices" and they included choice of examples, teachable moments, modeling, questioning, use of a variety of representations, and choice of task. In this paper, teachers' choice of examples is specifically discussed along with the impact this choice has on students' construction of understanding.

Theoretical Framework

Constructivism

The basic tenet of constructivism is that the learner constructs his/her own knowledge; each learner constructs a unique mental representation of the material to be learned and the task to be performed, selects information perceived to be relevant, and interprets that information on the basis of his or her existing knowledge (Shuell, 1996). The process is an active one and according to Shuell (1996) the most important determiner of what is learned. The construction of an idea will therefore vary from individual to individual even with the same teacher and within the same classroom (Van de Walle, 2007). The teacher's role is to ensure that students engage with the material to be learned and particularly to foster the connections between both different areas of mathematics and previous mathematics learning. The connectionist teachers identified in the Askew et al., (1997a) study were found to hold beliefs that supported this premise, including the need to explicitly recognise and work on misunderstandings (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997b).

Teacher Knowledge

In order for a teacher to practice within a constructivist paradigm, knowledge of the subject matter being taught, along with knowledge of the pedagogical principles needed to impart this knowledge to students, is required. There is a general lack of agreement over what exactly teachers need to know to teach mathematics (Hill, Schilling, & Ball, 2004), but teachers should possess a sufficient depth of understanding in order to communicate what is essential about a subject and be able to impart alternative explanations of the same concepts or principles (Shulman, 1987). Knowing more mathematics, however, does not ensure that one can teach it in ways that are meaningful for students (Mewborn, 2001). Individuals may have well-developed common knowledge, yet lack the specific kinds of knowledge needed to teach it (Hill et al., 2004). Pedagogical content knowledge (PCK) (Shulman, 1987) is of special interest because it represents "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). PCK involves preparation, representation of ideas and instructional selections from an array of teaching methods and models (Shulman, 1987) and as a mathematics teacher, "one needs to know the location of each piece of knowledge in the whole mathematical system, its relation with previous knowledge" (Ma, 1999, p. 115). The study of teachers' PCK has been the focus of recent research in mathematics education (e.g., Baker & Chick, 2006; Southwell, White, & Klein, 2004) and the highly successful

Cognitively Guided Instruction (CGI) program (Carpenter, Fennema, & Franke, 1996) focused on developing teachers' PCK through the provision of a framework that teachers could use to represent and explain a subject to make it comprehensible. Ma's (1999) comprehensive study found considerable differences in both the subject knowledge and PCK between Chinese and American teachers and used the term "profound understanding of fundamental mathematics" (PUFM) to define understanding a topic with depth. She argued that elementary mathematics is not a simple collection of disconnected numbers facts and algorithms, and therefore elementary teachers require PUFM in order to approach a topic in multiple ways, supporting the findings by Askew et al., (1997a), which indicated that the highly connectionist teachers were the most effective teachers of numeracy.

Choice of Example

One instructional strategy that teachers can use to help students construct meaning and one that plays a central role in the learning of mathematics is the use of examples. Examples may include illustrations of concepts and principles, contexts that illustrate or motivate a particular topic in mathematics and particular solutions where several are possible (Watson & Mason, 2002). Because examples are chosen from a range of possibilities (Watson & Mason, 2002), teachers need to recognise that some examples are "better" than others (Huckstep, Rowland, & Thwaites, 2003). A good instructional example is one which is transparent to the learner, helpful in clarifying and resolving mathematical subtleties and generalisable (Bills, Dreyfus, Mason, Tsamir, Watson, & Zavlavsky, 2006). Bills et al., (2006) maintain that the specific representation of an example or set of examples and the respective focus of attention facilitated by the teacher, have bearing on what students notice, and consequently on their mathematical understanding. Inappropriate examples can lead to a construction of understanding that was not the intention of the teacher. For example, when teaching analogue time to students, Huckstep et al. (2003) noticed a pre-service teacher using the example of "half past six" to demonstrate "half past". When the students she was teaching were subsequently asked to show "half past seven" on their clocks, one child put both hands on the "7". As the authors note, of the twelve possible examples available to exemplify "half past", "half past six" is arguably the least helpful (Huckstep et al., 2003).

Clearly the extent to which an example is transparent or useful is subjective, requiring the teacher to offer learning opportunities that involve a large variety of "useful examples" (Bills et al., 2006, p. 9). Ball (1990) questioned her own choice of examples when teaching fraction concepts to a third grade class. Presenting a scenario involving sharing a dozen cookies among family members appeared to be a legitimate example, based on a context familiar to students. Ball (1990) however, found problems with the social and cultural appropriateness of her choice, and the fact that the problem entailed cookies encouraged the use of a circle representation, making the drawing of equal parts inside the circle technically difficult. Similarly, Askew (2004) found that pupils will always interpret classroom tasks in the light of their previous experiences and that, "however carefully a teacher sets up a task, one cannot assume that the individual pupils' interpretations of that task … are either similar to each other's, or fit with the activity expectations of the teacher" (p. 74).

Methodology

A case study approach (Stake, 1995) was used to document the numeracy practice of three teachers. The researcher observed and videotaped between four and seven numeracy lessons (one each week) for each teacher. The transcripts of these lessons were then analysed, initially used the "principles of practice" and "observable teaching behaviours" to identify instances of occurrence. Data analysis was flexible, however, and allowed for other themes to emerge. Lesson and interview transcripts were analysed manually and instances of particular behaviours highlighted. Observable numeracy practices, including choice of examples, were identified and analysed. A lesson transcript, for example, may have included six instances where the teacher chose examples. Each of these examples was then examined for effectiveness in terms of its transparency and generalisability (Bills et al., 2006). In relation to this paper, the following research questions were identified:

- What is the nature of the examples chosen by teachers in the study?
- To what extent are these examples useful in students' construction of understanding?

It must be acknowledged that although the researcher attempted to evaluate the appropriateness of the examples based on classroom observations and her own pedagogical knowledge, it was not possible to ascertain whether or not the example was perceived to be equally appropriate (or not appropriate) for all participants. Three teaching episodes in which examples were used to demonstrate and develop strategies and concepts are discussed.

Results and Discussion

Problem Solving Examples

In the first lesson excerpt described, the teacher, Sue, introduced the problem solving strategy of "guess and check"; it was one in a series of lessons based on problem solving observed by the researcher. The whole class of grade 5/6 students was seated on the floor in front of Sue. Two examples were presented to the class, with the emphasis being on using a table to record the guesses. The researcher interpreted the teacher's intention as being primarily to introduce the guess and check problem solving strategy and then providing students with an efficient method of recording their working out through the use of a table. Students took turns to volunteer their "guesses" and showed their working out on the whiteboard using a table. The first example presented to the class was:

> Jenny collected 45 stickers over a 5 day period. Each day she was given 3 more stickers than the day before. How many was she given each day?

The example was interpreted by the researcher as being a good example in that guess and check was an appropriate strategy to be used and following some clarification as to what the problem was actually asking, students were able to use the strategy of guess and check correctly to eventually solve it. They also adjusted their "guesses" accordingly, based on the information gained from other students' attempts. The drawing of a table initially caused confusion for at least one student, who volunteered to draw up a table on the whiteboard and actually drew a dining table with four legs, but subsequent modeling by students established what the teacher intended by "make a table".

The second example involved larger numbers and although it was set out in a table, and could have been solved by using guess and check (it could also have been solved by working backwards), the larger numbers made it more difficult for children:

> A family set out on a 5-day trek. Each day they traveled 50 kilometres less than the day they had before. Total distance that they traveled was exactly 1500 kilometres. How far did they travel each day?

The following exchange highlights some of the difficulties posed by the numbers:

> Tr:[1] All right Mandy, have a go.
>
> M: (goes to board) I thought it was, they started on 900.
>
> Tr: OK, so you're going to say they traveled 900 on Monday.
>
> Tr: 900 – that says 90 (Mandy has written 90 in the first column – she then adds another 0).
>
> Tr: OK, so how far would they travel on Tuesday?
>
> M: 850.
>
> Tr: OK.
>
> (Mandy writes down 850)
>
> Simon: They have to get to 0.
>
> Tr: Do we have to get to 0 Simon? What does it have to add up to? What do they all have to add up to?
>
> S: Oh, 1500.
>
> Tr: So looking at that, who's going to have another guess and see what they can come with? Randall? Just use that (referring to table drawn) and put a line down. So which one are you going to use Randall?
>
> R: 200.
>
> Tr: 200, all right.
>
> (Randall starts filling in table, beginning with 200)
>
> Tr: So they're not traveling – whoops – they're not traveling anywhere on Friday? They're going to stay at home. OK, so is that going to add up to 1500? Is it Randall? What does it add up to? 3, 4, 500?

As the process of "guess and check" was being introduced as a new strategy, it was unfortunate that the inclusion of larger numbers in the second problem created confusion and detracted from the modeling or consolidation of the "guess and check" process. The students did not have a written copy of the problem to refer to and Sue later reflected that this would have been beneficial.

Following the sharing of these two problems, students were issued with problems to complete individually. One of the problems involved identifying how many lizards and spiders there would be if one counted 60 legs and 10 heads – this may have been a preferable example to model with the whole class as it involved smaller numbers and arguably better suited the "guess and check" process. Although it is not possible to generalise that the whole class understood the process and used a table to record their guesses and checks, Figure 1 shows a typical response to the problem and indicated that this student did construct the understanding intended.

[1] Tr. refers to teacher throughout

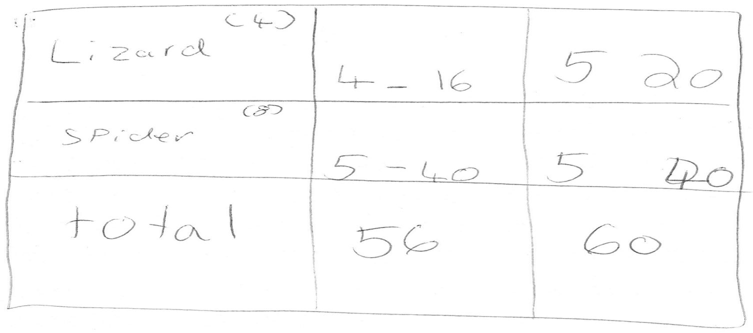

Figure 1. Student's use of table to record guess and check strategy.

Sue's lesson highlighted the need for teachers to consider their choice of example in the context of students' previous mathematical experiences. If the aim was for students to construct an understanding of the guess and check process, then it was unfortunate that some students may have been excluded from the process because of their lack of confidence in operating with larger numbers. Examples need to be selected that are suitably challenging and motivating enough in order to engage students, yet still provide for the desired construction of understanding. The teacher's judgement is vital here, with appropriate selection likely being influenced by both the teacher's content and pedagogical content knowledge.

Percentage Examples

The following lesson details a second teacher, Ronald's, selection of examples with relation to percentages. This was the first "formal" introduction that the grade 5/6 had to percentages and it followed on from work on decimals and fractions. Following a general brainstorming discussion about percentages, students worked in small groups to identify where, why and how percentages were used. Several authentic examples were then shared and connections were made with real life, such as sport and discounts, and links were made within mathematics to decimals and fractions. Ronald then moved on to teach how to calculate percentages explicitly. The initial example selected was 20% of 100, and this was recorded on the board:

Tr: Just looking at that, can somebody tell me what 20% of 100 might be?

N: 20.

Tr: Why do you think the answer is 20 Nigel?

N: 'Cause it's out of 100.

Ronald reminded students about the process they used to multiply fractions and related this to the process used to find percentages of numbers using the above example. This example was worked through with the whole class with a variety of students contributing answers. The example chosen was deemed to be appropriate in that the numbers were 'friendly' to work with and the process could be used to demonstrate that the same answer was obtained as the original response. Further examples were also given, including 10% of 90 and 5% of $5.00 and the process was worked through again with the whole class. Again these examples were considered appropriate as they included a diversity of numbers yet were still reasonably straightforward to operate with (the 5% of $5.00 had the potential to be problematic, but the students appeared comfortable with the division of decimals).

Through the selection of the examples and the order in which they were presented the teacher provided scaffolding for the students to construct their understanding of the process. The use of different amounts and the money context may have also counteracted students' inclination to form the construction that percentages could only be calculated with amounts of 100.

Students then worked individually to calculate the example, 10% of 110. The choice of this example demonstrated that percentages could involve numbers more than 100, yet the numbers were easy to operate with. Students appeared to be comfortable with the process and the solution was again shared with the whole class. Ronald then wrote the following examples on the board which the students were expected to complete individually: 25% of 100, 15% of 200, 30% of 96, 60% of 110 and 24% of $48; 30% of 96, and 24% of $48 proved problematic for some students and Ronald later reflected on his choice of examples:

> They picked it up really quite quickly – once I did a couple of more complicated ones on the board, probably three quarters of the class were picking it up, and that one quarter who were still struggling, they were struggling with the numbers not with the actual process – I probably stuffed up with the last example that I used – I put 24% of something, when I should have put 25% - that's just one of those errors that can just happen, but in another way it was an advantage because it showed me those kids who would persevere through something when they come to a problem that wasn't straight forward … and it also showed me the limitations of some kids at this point in time

Ronald's choice of examples generally indicated he possessed a strong content knowledge of mathematics and PCK – the examples were mostly appropriate in terms of the numbers involved and the order in which they were presented provided for scaffolding of students' understanding to occur. He recognised that 24% was not a good example, but then interpreted it as a positive and used it as a subsequent teaching point. The excerpt also illustrates the value of using a variety of examples and Ronald's awareness of the links between different aspects of the mathematics curriculum (Askew et al., 1997b).

Decimal and Money Examples

The next lesson differs from the previous ones in that it documents a teaching episode involving a group of four grade 8 students. The lesson was conducted by a specialist mathematics teacher and the small group focus allowed for more interaction between teacher and students to occur than probably would under normal classroom conditions. The aim of the session was to gauge where students were at in terms of their understanding of place value involving whole numbers, then expand on this knowledge to include decimals. The students and teacher were seated around a table and the students had access to paper, pens and bundling sticks. Following a discussion and some demonstration involving bundling the sticks into groups of ten and a hundred, and feeling confident that the students could accurately represent a four-digit number using the materials, Jeff asked one of the students, John, to cut one of the sticks into parts to represent tenths. As he began randomly cutting the stick, a discussion occurred on whether or not the parts needed to be equal. To demonstrate this point, Jeff used the example of the bundling sticks and stated,

> OK, suppose I ask you this – see that number there 5345 – now with that 5, would it be all right do you think if we had 1000 in this pack and 995 in another pack and a group of 1500 in another pack, or is it important that all the group sizes are the same when we write a number like that?

John still was not convinced and stated that, "It doesn't matter what size the things are as long as you've got ten of them there."

Jeff continued to question John and tried to use the materials to demonstrate that the size of the pieces were important, but John remained confused. Jeff then decided to use the example of money and this choice proved to be quite problematic. Lampert (1989, as cited in Ball, 1990) argues that money may provide a useful familiar context to develop students' understanding of decimals, but the particular way in which this example was presented led to further confusion.

> Tr: If you were working in the work place and you were getting paid … you guys might get paid \$5.00 an hour and let's say you worked three hours for \$15.00 one night – now let's say, Cara, you work Wednesdays and Sarah worked Tuesdays and Sarah was actually getting more money than you on Tuesdays because \$15.00 actually meant a bit more than \$15.00 on Wednesdays – would you be very happy with that?

Jeff was using a context that he thought students would relate to, but did not anticipate the following response from Cara, "But some days, like Saturday and Sunday you do get paid more because it's like the weekend."

Jeff immediately recognised the logic behind this, and stated that,

> "Yeah, well this is an example of how my question hasn't worked there because the understanding I'm trying to get out of you – you've actually gone off on another whole track…"

The students still seem confused but agreed that it should be worth the same each day. Although not mentioned by any of the students, further confusion could have been created by the use of this particular example, as the value of the dollar does fluctuate and can indeed be seen to be worth more on a particular day as it varies in exchange rates for different countries.

The money example further proved problematic when Jeff asked students to write down \$1.05. Although Cara wrote the amount correctly, Adam wrote it as \$1.5. After some discussion and when there was no general agreement in the group about which was correct, Jeff returned to the bundling sticks:

> Tr: So (if we say) this is a dollar coin (holds up one bundling stick), this is a ten dollar note (holds up a bundle of ten sticks) and this is a hundred dollar note (holds up bundle of 100) and pretend just for today that we have a thousand dollar note (holds up the bag of 1000 sticks), so what would that bit there be worth (points to one of the chopped up pieces of bundling stick)?
>
> J: A five cent coin.
>
> Tr: A five cent coin – and how do you know that?
>
> J: Because five cents is the smallest and that (piece) is smaller than the rest of them.

John's answer shows a clear example of pupils interpreting information in light of their previous experiences (Askew, 2004); because the one cent coin is no longer in circulation, his response is quite a logical one. Jeff then reminded the students that we did actually once use one cent coins (although the piece of stick would actually represent ten cents) and made the comment that the five cent coin will probably be the next coin to go, making ten cents the smallest denomination. The exchange illustrates that again, although money is a common example to use when teaching decimals, it was proving to be too abstract for these students to construct a meaningful understanding of what the numbers on the right of the decimal point actually represented. Furthermore, during the plenary session when Jeff was encouraging the students to reflect on what they had learned from the lesson, one of the students, Sarah, responded that, "I learnt that five cent pieces will go…"

Jeff's reflection on the lesson revealed that he had to abandon the plan he originally made for the lesson because of the students' lack of conceptual understanding about the place value system, but that the lesson was useful in that it identified the misconceptions that they did hold. He stated:

> If you really believe in the constructivist view of learning, then it's really about the students' understanding and that's where the questioning is so important and you have to keep questioning – it's like the teacher is a mathematical doctor and the questions are like a scalpel and you keep probing at what it is they're picking up and leading them towards that desired understanding so they go, yes, I see what you're getting at.

Conclusions

In order to help students develop mathematical understanding, the teacher must select examples that enable students to construct accurate knowledge about the concepts presented. Effective teaching for numeracy involves many aspects, but through the careful consideration of selecting which examples to choose and then reflecting on their effectiveness, teachers can reduce the likelihood of students forming misconceptions about important mathematical concepts. The treatment of examples presents the teacher with a complex challenge and the specific choice and manner of working with examples can either facilitate or impede learning (Bills et al., 2006). This paper has provided descriptions about the types of examples teachers select and the way in which students construct understanding, based on these examples. It supports the findings of other research (e.g., Huckstep, et al., 2003; Ball, 1990) and acknowledges the role that teacher content knowledge and PCK play in selecting and presenting these examples. The discussion of Jeff's lesson also shows that even teachers considered to hold both sound content knowledge and PCK, can still select examples that do not lead to accurate construction of meaning. Further research may be needed to look at the link between teachers' PCK and the selection of good examples, along with research on comparing teachers' and students' perceptions of what constitutes a good example and documentation of individual students' interpretations of the same example.

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